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TECHNICAL MEMORANDUMS

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

No. 1038

THE PERFORMANCE OF A VANELESS DIFFUSER FAN

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U. S. NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS
LABORATORY
Langley Field, Va.

Washington
December 1942



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The present paper is devoted to the theoretical and experimental investigation of one of the stationary elements of a fan, namely, the vaneless diffuser. The method of computation is based on the principles developed by Pfleiderer (Forschungsarbeiten No. 295). The practical interest of this investigation arises from the fact that the design of the fan guide elements - vaneless diffusers, guide vanes, spiral casing - is far behind the design of the impeller as regards accuracy and reliability. The computations conducted by the method here presented have shown sufficiently good agreement with the experimental data and indicate the limits within which the values of the coefficient of friction lie.

NOTATION

b width of flow section, m

c velocity, m/sec

d hub diameter, m²

F area, m²

H pressure head, mm water

$$K_{\alpha} = \frac{1}{\sin \alpha} \left[1 + \frac{1}{\tan^2 \alpha} \right]$$

$$K_{\lambda} = \frac{\lambda}{2b}$$

$$K = K_{\alpha} K_{\lambda} \rho / 2$$

M moment, kg m

*Report No. 224, of the Central Aero-Hydrodynamical Institute, Moscow, 1935.

N power, hp
 R variable radius, m
 ds length of flow path, m
 U perimeter, m
 Q air discharge, m^3/sec
 $Q' = Q_{sec}/b$
 ω = angular velocity, $1/sec$
 α angle formed by the absolute velocity with its tangential component
 η efficiency
 λ friction coefficient
 ρ density, $kg\ sec^2/m^4$
 Γ circulation, m^2/sec
 ΔH_r pressure loss due to friction, mm water
 ΔH_{imp} pressure loss due to impact, mm water

Subscripts

a_a parallel to the axis
 a referring to the value of the variable at the impeller outlet
 3 referring to the value of the variable at the diffuser inlet
 a referring to the value of the variable at the diffuser outlet
 h hydraulic
 m in the meridian plane
 u in the tangential plane
 d dynamic

st static
th theoretical

I. GENERAL CONSIDERATIONS

The fundamental part of every fan whether centrifugal or axial (also turbofans and compressors) is the rotating impeller (or rotor) since it is in the latter, only that the energy increase of the air passing through the fan occurs. However, no matter how good the impeller design from the aerodynamic point of view, the flow at the impeller exit needs to be modified for the following two reasons: In the first place, for air delivery to a pipe system or to a second stage, in a multistage machine, it is necessary to concentrate the flow in a given direction.* In the second place a considerable portion of the energy of the flow at the exit is in the form of kinetic energy, which in the absence of guide vanes or a spiral casing is entirely lost, the efficiency then assuming inadmissibly low values. For these reasons it is generally found desirable to increase the static pressure at the expense of a reduced velocity pressure of the air stream issuing from the impeller and this is done in the stationary elements of the fan. When the flow passes through these elements the total energy of the flow necessarily decreases since it is partly dissipated by the losses but the gain in the value of the static pressure entirely justifies a certain decrease in the total pressure.

The vaneless diffuser is not the only solution of the problem. In the case of multistage machines, for example, a vaned diffuser is often employed which in a number of cases gives better results than the vaneless diffuser. The present article is concerned with the investigation of the vaneless diffuser and the performance of a vaned diffuser will be mentioned below only for comparison with the vaneless diffuser.

A very large number of centrifugal fans consist simply of the impeller followed immediately by the spiral casing. The more or less considerable value of the impact losses at the inlet to the spiral casing appears as a special feature

*A rare exception is the case of a dispersed air discharge

of almost all such designs. This makes it necessary, in a number of cases, to introduce between the impeller and casing, an intermediate element, namely, the diffuser. In certain cases, the transformation of the kinetic energy in the diffuser is so complete as to permit dispensing entirely with the spiral casing and replacing it by a symmetric circular channel.

It is obvious that in a single-stage machine a diffuser is of advantage only in those cases where the losses in it plus the impact losses at the inlet to the spiral casing (from diffuser) are considerably less than the impact losses from the impeller directly connected to the casing.

The vaneless diffuser consists of two parallel planes between which the air from the impeller flows (fig. 1). The computation of the diffuser reduces to determining at a given value of the static pressure and magnitude and direction of the velocity at the entrance to the diffuser, the change in the velocity vector while passing through the diffuser, and the new values of the static pressure and velocity at the diffuser exit.

In order not to go beyond the scope of this paper we shall consider the data characterizing the flow at the impeller exit and the magnitude of the power required by the impeller as known.

II. MODE OF OPERATION OF THE VANELESS DIFFUSER AND FLOW PATH

Before taking up, in detail, the investigation of the flow in the vaneless diffuser with the aid of a quantitative analysis we shall consider the broad features of the flow process.*

The flow path in the vaneless diffuser is determined by the aerodynamics of the impeller and its operating conditions. If the direction of the velocity at the diffuser inlet is sufficiently near radial, as is the case with backward curved blades and large air discharge (fig. 2), the flow passes through the diffuser rapidly; the length of the

* The hydraulic impact at the diffuser inlet is not here considered since the corresponding losses, determined individually, enter the general magnitude of the losses independently of the diffuser computation. (See Impact Losses.)

flow path is small and the pressure losses in it are also small (we are here considering, of course, not the absolute but the relative value of the losses at the impeller exit). If the direction of the flow velocity in the diffuser is nearly tangential to the circumference, as is the case with a small air discharge independently of the blade profile (fig. 3), the flow during a small displacement in the radial direction will simultaneously be displaced for a considerable distance in the tangential direction. The flow path, a spiral, is lengthened and the losses in the diffuser become very considerable. The suitability of a vaneless diffuser as compared with guide vanes, should in this case be checked by a comparison computation.

The investigation here undertaken connects the values of interest with the magnitude and direction of the velocity, which we shall now determine.

The velocity, C , of a certain point of the flow at the variable distance, R , from the axis may be resolved into two components (fig. 4) of which the first, C_m , is directed along the radius and the second, C_u along the tangent to the circle of radius, R and lying in the plane of the circle. The two components C_m and C_u are denoted, respectively, as the meridional (or radial) and tangential components. The latter component produces a rotation of the flow and may, therefore, be denoted as the rotational component.

We note that

$$C_m^2 + C_u^2 = C^2 \quad (1)$$

The radial velocity C_m , for the case where the flow fills out the width of the diffuser, may be directly determined from the continuity equation

$$C_m = \frac{Q_{sec}}{\pi D b} \quad (2)$$

Let us now consider how the tangential velocity C_u is determined. The mass of air flowing through per second is ρQ_{sec} ; the momentum $\rho Q_{sec} C_u$; the momentum $\rho Q_{sec} C_u R$; the change in the moment of momentum (referred to Q_{sec}) is equal to the moment of the applied external forces M :

$$M = \Delta (\rho Q_{\text{sec}} C_u R)$$

For an ideal flow when the moment of the external forces is equal to zero the above formula indicates the constancy of the moment of momentum for the entire diffuser:

$$\rho Q_{\text{sec}} C_u R = \rho Q_{\text{sec}} C_{3u} R_3$$

or

$$R C_u = \text{constant} \quad (3)$$

As regards the magnitude $C_{3u} = C_{2u}$ it may be found not only by the method of successive computation of the velocities in the impeller but also directly from the values of the hydraulic power* obtained experimentally.

Thus

$$N_h = \frac{H_{Th} Q_{\text{sec}}}{75^{**}}$$

$$H_{Th} = \rho U_2 C_{2u}$$

Hence

$$N_h = \frac{\rho U_2 C_{2u} Q_{\text{sec}}}{75^{**}}$$

and

$$C_{2u} = \frac{N_h 75^{**}}{\rho U_2 Q_{\text{sec}}}$$

$$C_{3u} = C_{2u} \quad (\text{since } R = R_2)$$

*The hydraulic power is the entire power after deducting N_0 , the power expended in friction of the disks and eddy formation.

** Where U_2 = tip speed of wheel (m/sec). (Actual conversion figure of kg m/sec to hp is 76.1, not 75.)

Knowing the law of motion of the flow in the directions of C_m and C_u it is possible to find the trajectory of the flow in the plane of the diffuser. We have

$$C_m = \frac{Q_{sec}}{\pi D b} \text{ and } C_u = \frac{\Gamma}{2\pi R}$$

Then

$$\frac{C_m}{C_u} = \frac{Q_{sec}}{b\Gamma} = \frac{Q'}{\Gamma} = \text{constant} \left(\text{where } Q' = \frac{Q_{sec}}{b} \right)$$

Denoting by α the angle formed by the velocity vector with its tangential component, the equation $C_m/C_u = \text{constant}$ assumes the form

$$\tan \alpha = \text{constant}$$

and

$$\alpha = \text{constant}$$

From this it follows that the trajectory of the ideal flow in the plane of the diffuser is a logarithmic spiral whose equation may be written in another form

$$R = R_3 e^{\frac{Q'}{\Gamma} \alpha}$$

III. RESISTANCE IN THE VANELESS DIFFUSER*

1. Distribution of the Losses

During the passage of the air through the diffuser, friction occurs at the diffuser walls. The frictional force constituting the resistance produces a certain drop in the pressure, the pressure drop per unit distance evidently decreasing on approaching the outlet (due to the decrease in the velocity).

Let us consider the resistance of an infinitely small flow element for a path element dS (fig. 4) to which corresponds a pressure loss

$$dH_r = \lambda \frac{dS}{D_h} \rho \frac{C^2}{2} \quad (6)$$

where D_h is the hydraulic diameter. The latter, as is known, is equal to 4 times the hydraulic radius, which in turn is the ratio of the flow cross section to its perimeter; in our case

$$D_h = 4 \frac{F}{U} = 4 \frac{b dy}{2 dy} = 2b$$

(for the perimeter the value $2dy$ is taken and not $2dy + 2b$ since along the line b the element touches the neighboring elements which are also in motion).

*The computation scheme and the derivation of the fundamental equation

$$\ln \left[\frac{1 + \sin \alpha}{\cos \alpha} \frac{\cos \alpha_3}{1 + \sin \alpha_3} \right] = \frac{\lambda}{4b} (R - R_3)$$

are those of Pfleiderer. The further adaptation and simplification of the equation with the object of applying it to practical cases is the fundamental object of this paper. In order to bring out more clearly the physical aspect of the process the derivation of the fundamental equation is somewhat modified.

$$dH_r = \lambda \frac{dS}{2b} \rho \frac{C^2}{2} = \lambda \frac{dS}{2b} \rho \frac{C_m^2 + C_u^2}{2} = \lambda \frac{dS}{2b} \rho \frac{C_m^2}{2} + \lambda \frac{dS}{2b} \rho \frac{C_u^2}{2}$$

Denoting $\lambda \frac{dS}{2b} \rho \frac{C_m^2}{2}$, that is, the losses depending on the radial velocity by dH_{rm} , and $\lambda \frac{dS}{2b} \rho \frac{C_u^2}{2}$, that is, the losses depending on the tangential velocity by dH_{ru} , we obtain

$$dH_r = dH_{rm} + dH_{ru}$$

In overcoming the resistance along its path the flow loses part of its total pressure, the pressure loss occurring at the expense of both the static and dynamic pressure drops.

Let us consider the mechanism of the losses associated with the radial and tangential velocity components C_m and C_u . The work of a flow element for an infinitesimal element of the path dS is entirely expended in overcoming the friction, (i.e. $\eta = 1$). For this case the usual formula

$$N = \frac{Q_{sec} h}{75 \eta} \text{ (in hp) or } N = \frac{Q_{sec} h}{\eta} \text{ (in kg m/sec)}$$

($h = \text{kg/M}^2$ or pressure) h is expressed as pressure drop. It assumes the form

$$dN = Q_{sec} dH_r$$

The work of the force in overcoming the friction during the time dt is

$$Q_{sec} dH_r dt$$

If this work is done over the path DS the magnitude of the force is

$$\frac{Q_{sec} dH_r dt}{dS}$$

Substituting in the above formula the expression

$\lambda \frac{dS}{2b} \rho \frac{C^2}{2}$ for dH_r we obtain

$$\frac{Q_{sec} dH_r dt}{dS} = \lambda Q_{sec} \frac{dt}{2b} \rho \frac{C^2}{2}$$

The moment of the force about the axis is equal to the change per second of the momentum of momentum:

$$\lambda Q_{sec} \frac{dt}{2b} \frac{\rho C^2}{2} R \frac{C_u}{C} = \rho Q_{sec} R (dC_u)''$$

where RC_u/C is the lever arm of the force $(dC_u)''$ is the loss in velocity due to friction. Dividing through we obtain

$$(dC_u)'' = \frac{\lambda}{4b} dt C C_u$$

Noting that

$$C = \frac{dS}{dt}$$

$$C_m = \frac{dR}{dt}$$

then

$$(dC_u)'' = \frac{\lambda}{4b} dR \frac{C}{C_m} C_u$$

Multiplying both sides of the equation by $C_u \rho$ we obtain

$$C_u (dC_u)'' \rho = \frac{\lambda}{4b} dR \frac{C}{C_m} C_u^2 \rho$$

Since $C/C_m = 1/\sin \alpha$

$$C_u (dC_u)'' \rho = \frac{\lambda}{2b} \frac{dR}{\sin \alpha} \rho \frac{C_u^2}{2} = \frac{\lambda}{2b} dS \rho \frac{C_u^2}{2}$$

But the right side of the equation is equal to dH_{ru} and the left side is the differential of the dynamic pressure corresponding to the tangential velocity component $\rho C_u^2/2$. Hence

$$dH_{ru} = d \left(\rho \frac{C_u^2}{2} \right)''$$

that is, the friction loss associated with the velocity C_u is the loss of the dynamic head corresponding to the rotational velocity component.

The losses in the radial direction do not occur at the expense of the dynamic pressure since the velocity C_m is determined by the cross section independently of the force of friction. These losses are therefore at the expense of the pressure, that is, the static pressure head. Thus the static pressure loss, representing the difference between the total and dynamic pressures is equal to

$$\lambda \frac{dS}{2b} \rho \frac{C^2}{2} - \lambda \frac{dS}{2b} \rho \frac{C_u^2}{2} = \lambda \frac{dS}{2b} \rho \frac{C_m^2}{2} = dH_{rm}$$

2. Determination of the True Exit Angle of the Flow

We have found

$$\rho Q_{sec} C_u R = \text{constant}$$

or

$$C_u R = \text{constant}$$

This expression holds only in the ideal frictionless case. In the actual process, however, there is an additional decrease in the velocity due to friction, and the actual value of C_u will be less than is obtained from the above condition.

A closer investigation will now be made to determine the actual value of C_u at various points of the casing. In the absence of friction the drop in the rotational velocity component (dC_u) over an infinitely small path dS may be readily obtained by differentiating the expression $C_u R = \text{constant}$:

$$(dC_u)' R + C_u dR = 0$$

or

$$(dC_u)' = - \frac{C_u dR}{R}$$

Due to friction there is an additional loss at the expense of the dynamic pressure given by

$$dH_{ru} = d \left(\rho \frac{C_u^2}{2} \right) = C_u (dC_u)' \rho$$

Above we had obtained

$$C_u (dC_u)'' \rho = \lambda \frac{dR}{2b \sin \alpha} \rho \frac{C_u^2}{2}$$

Dividing, we obtain

$$(dC_u)'' = \lambda \frac{dR}{4b \sin \alpha} C_u$$

The total drop in the tangential velocity is

$$dC_u = (dC_u)' + (dC_u)'' = - \frac{C_u}{R} dR - \lambda \frac{dR}{4b \sin \alpha} C_u$$

Multiplying both sides by R we obtain

$$dC_u R + C_u dR = - \lambda \frac{dR}{4b \sin \alpha} C_u R$$

or

$$\frac{d(RC_u) \sin \alpha}{RC_u} = - \lambda \frac{dR}{4b}$$

We note that

$$(a) \quad RC_u = \frac{RC_m}{\tan \alpha} = \frac{R Q_{sec}}{2\pi R b \tan \alpha} = \frac{Q_{sec}}{2\pi b \tan \alpha}$$

$$(b) \quad d(RC_u) = \frac{Q_{sec}}{2\pi b} d\left(\frac{1}{\tan \alpha}\right)$$

$$(c) \quad \sin \alpha = \frac{\tan \alpha}{\sqrt{1 + \tan^2 \alpha}}$$

Substituting all these values for RC_u , $d(RC_u)$ and $\sin \alpha$ in the last formula, we obtain

$$\frac{\frac{Q_{sec}}{2\pi b} d\left(\frac{1}{\tan \alpha}\right) 2\pi b \tan^2 \alpha}{Q_{sec} \sqrt{1 + \tan^2 \alpha}} = - \frac{\lambda dR}{4b}$$

Dividing through leads to the differential equation

$$\frac{d\left(\frac{1}{\tan \alpha}\right) \tan^2 \alpha}{\sqrt{1 + \tan^2 \alpha}} = - \frac{\lambda dR}{4b}$$

whose solution is

$$\ln \left(\frac{1 + \sin \alpha}{\cos \alpha} \frac{\cos \alpha_3}{1 + \sin \alpha_3} \right) = \frac{\lambda}{4b} (R - R_3) \quad (7)$$

or

$$\frac{1 + \sin \alpha}{\cos \alpha} \frac{\cos \alpha_3}{1 + \sin \alpha_3} = e^{\frac{\lambda}{4b} (R - R_3)} \quad (8)$$

In any concrete case when the friction coefficient, the radii and width of the casing are known, the expression at the right side may be easily determined. Denoting it by K_1 we obtain for the determination of the exit angle from the vaneless diffuser the following expression:

$$\frac{1 + \sin \alpha_a}{\cos \alpha_a} = \Phi_{\alpha a} = K_1 \frac{1 + \sin \alpha_3}{\cos \alpha_3}$$

Elementary transformations of the expression $\frac{1 + \sin \alpha_a}{\cos \alpha_a} = \Phi_{\alpha a}$ lead to the quadratic equation

$$\sin^2 \alpha_a + \frac{2}{\Phi_{\alpha a}^2 + 1} \sin \alpha_a + \frac{\Phi_{\alpha a} - 1}{\Phi_{\alpha a} + 1} = 0$$

One of the roots of this equation; namely, $\sin \alpha_a = -1$, is a trivial solution corresponding to the case, impossible for the centrifugal machine where the radial velocity is directed toward the center.

The second root is

$$\sin \alpha_a = \frac{\Phi_{\alpha a}^2 - 1}{\Phi_{\alpha a}^2 + 1} \quad (9)$$

In the above equation the angle α_a is a real angle which the absolute velocity of flow at the diffuser outlet makes with the tangential component. Hence

$$C_{au} = \frac{C_{am}}{\tan \alpha_a} \quad (10)$$

where

$$C_{am} = \frac{Q_{sec}}{2\pi R_a b}$$

All of the above computations may be essentially simplified as a consequence of the very interesting fact, namely, that the change in the angle α is a linear function of the change in diameter (reference 1).

This law may be easily derived analytically. We have already obtained the equation

$$\ln \left(\frac{1 + \sin \alpha}{\cos \alpha} \frac{\cos \alpha_3}{1 + \sin \alpha_3} \right) = \frac{\lambda}{4b} (R - R_3) \quad (11)$$

or

$$\frac{\lambda R}{4b} = \ln \frac{1 + \sin \alpha}{\cos \alpha} - \ln \frac{1 + \sin \alpha_3}{\cos \alpha_3} + \frac{\lambda R_3}{4b}$$

Expanding this expression in a Taylor series

$$\begin{aligned} \frac{\lambda R}{4b} = & -\ln \frac{1 + \sin \alpha_3}{\cos \alpha_3} + \frac{R_3 \lambda}{4b} + \ln \frac{1 + \sin \alpha_3}{\cos \alpha_3} + (\alpha - \alpha_3) \sec \alpha_3 \\ & + \frac{(\alpha - \alpha_3)^2}{1 \times 2} \sec \alpha_3 \tan \alpha_3 + \frac{(\alpha - \alpha_3)^3}{1 \times 2 \times 3} \sec \alpha_3 (1 + 2 \tan^2 \alpha_3) + \dots \end{aligned}$$

or

$$\begin{aligned} \frac{\lambda(R - R_3)}{4b} = & (\alpha - \alpha_3) \sec \alpha_3 + \frac{(\alpha - \alpha_3)^2}{1 \times 2} \sec \alpha_3 \tan \alpha_3 \\ & + \frac{(\alpha - \alpha_3)^3}{1 \times 2 \times 3} \sec \alpha_3 (1 + 2 \tan^2 \alpha_3) + \dots \end{aligned}$$

where α is expressed in radians. It is readily shown that in practical computations all terms except the first may be neglected. Thus assuming that $\alpha_a - \alpha_3 = 10^\circ$ (0.175 rad) $\alpha_3 = 45^\circ$ *

$$\frac{\lambda(R-R_3)}{4b} = 0.175 \frac{1}{0.71} + \frac{0.175^2}{1 \times 2} \frac{1}{0.71} + \frac{0.175^3}{1 \times 2 \times 3} \frac{1}{0.71} (1 + 2 \times 1.0^2) \dots$$

$$\frac{\lambda(R-R_3)}{4b} = 0.246 + 0.02 + 0.004 = 0.27$$

Retaining only the first term, 0.246, we obtain a deviation in 10 percent for $\alpha - \alpha_3 = 10^\circ$. This corresponds to an error of 1 percent. An error in the determination of the value of $\alpha_a - \alpha_3$ of the order of 10 percent corresponds to a considerably smaller error in the determination of the losses. The friction coefficient entering the computation is obtained, however, with a considerably smaller degree of accuracy.

As further investigations have shown the effect of the exit angle from the diffuser on the losses is so small that an error in the determination of this angle of even 5° is entirely admissible, so that the above error of the order of 1 percent is all the more admissible. Thus

$$\alpha_a = \alpha_3 + \frac{\lambda \cos \alpha_3}{4b} (R_a - R_3) \quad (12)$$

*With regard to our choice of the values of $\alpha_a - \alpha_3$ and $\alpha_3 = 45^\circ$, it should be noted that values of 10° for $\alpha_a - \alpha_3$ are practically not encountered; large values of $\alpha_a - \alpha_3$ correspond to a small value of α_3 and values of $\alpha_a - \alpha_3$ of the order of $1^\circ - 3^\circ$ correspond to large values of α_3 ($\alpha_3 > 30^\circ$). The assumed values of 10° for $\alpha_a - \alpha_3$ and 45° for α_3 , therefore, do not occur simultaneously and our example should give a very exaggerated result of neglecting the second and following terms of the series; all the more since these terms contain $(\alpha_a - \alpha_3)^2$, $(\alpha_a - \alpha_3)^3$, etc.

Formula (12) shows that the increase in the angle α is proportional to the increase in the radius R , or in other words, as a linear function of the radius of the diffuser. This law is of cardinal importance in the computation of vaneless diffusers since it makes it possible, knowing the entrance angle into the diffuser, to compute readily the exit angle and then, if required, to take intermediate values of α and construct the straight line $\alpha = f(R)$. We may remark, incidentally, that this curve may serve as an index of the magnitude of the losses since for large losses the angle α varies more rapidly.

3. Losses in the Vaneless Diffuser

Having determined the value of the angle α_n at the diffuser exit we may now proceed to determine the magnitude of the losses in the diffuser.

We have found

$$dH_{rm} = \lambda \frac{dS}{2b} \rho \frac{C_m^2}{2}$$

But

$$dS = \frac{dR}{\sin \alpha} \quad (\text{see fig. 4})$$

Hence

$$dH_{rm} = \frac{\lambda}{2b} \frac{dR}{\sin \alpha} \rho \frac{C_m^2}{2} \quad (13)$$

The losses associated with the tangential velocity component may be expressed by an equation of the same form as that above:

$$dH_{ru} = \lambda \frac{dS}{2b} \rho \frac{C_u^2}{2}$$

Substituting $dR/\sin \alpha$ for dS and $(C_m/\tan \alpha)^2$ for C_u^2 we obtain

$$dH_{ru} = \frac{\lambda}{2b} \frac{dR}{\sin \alpha} \rho \frac{C_m^2}{2} \frac{1}{\tan^2 \alpha} \quad (14)$$

Comparing (13) and (14), we note that

$$dH_{ru} = dH_{rm} \frac{1}{\tan^2 \alpha}$$

Adding the losses we obtain

$$dH_r = dH_{rm} + dH_{ru} = \frac{\lambda}{2b} \frac{1}{\sin \alpha} \left(1 + \frac{1}{\tan^2 \alpha} \right) \rho \frac{C_m^2}{2} dR$$

Denoting $\lambda/2b$ by K_λ and

$$\frac{1}{\sin \alpha} \left(1 + \frac{1}{\tan^2 \alpha} \right) \text{ by } K_\alpha \text{ (the value of the variable)}$$

we have

$$dH_r = K_\lambda K_\alpha \rho \frac{C_m^2}{2} dR \quad (15)$$

We may note the following: The expression denoted by K_α in a number of cases has at the end of the trajectory a value near the initial one. In these cases we may substitute a mean value for K_α in the computation. Substituting this value in formula (15) we obtain

$$\Delta H_r = \int_{R_3}^{R_a} dH_r = \int_{R_3}^{R_a} K_\lambda (K_\alpha)_m \rho \frac{C_m^2}{2} dR$$

where

$$C_m^2 = \left(C_{3m} \frac{R_3}{R} \right)^2$$

whence

$$\Delta H_r = \int_{R_3}^{R_a} K_\lambda K_{\alpha m} \rho \frac{C_{3m}^2}{2} R_3^2 \frac{dR}{R^2} = K_\lambda K_{\alpha m} \rho \frac{C_{3m}^2}{2} R_3^2 \left(\frac{1}{R_3} - \frac{1}{R_a} \right) \quad (16)$$

Thus for a small change in the value of $\sin \alpha$ (a change of the order of 10 percent and over, if the computations are approximate) the losses are determined by numerical computations without the aid of auxiliary graphs.

In those cases where the changes in $\sin \alpha$ are considerable, the flow trajectory may be divided into a number of intervals, the value of dH_r/dR computed for each interval and then graphically integrated: that is, the curve

$$\frac{dH_r}{dR} = f(R)$$

is constructed and the area

$$\Delta H_r = \int_{R_3}^{R_a} \frac{dH_r}{dR} dR$$

integrated under the curve (see fig. 6). The entire computation procedure is made clear by the example given below.

4. Impact Losses

If the active flow does not fill out the impeller cross-sectional width at the exit and also in the cases where the impeller width is less than the diffuser width, as is the most typical case, the impact losses at the entrance to the diffuser must be added to the frictional losses.

Simplifying, somewhat, the actual impact phenomenon the computation may start from the assumption that the flow at the diffuser entrance momentarily fills out the diffuser width, that is, the radial velocity C_{3m} at the entrance may be taken equal to $Q_{sec}/\pi D_3 b_3$ where $D_3 = D_2$. In correspondence with this assumption the impact losses are determined as

$$\Delta H_{imp} = \frac{\rho}{2} (C_{2m} - C_{3m})^2 \quad (17)$$

The computation above is based on the assumption that the vaneless diffuser does not change the entire kinematic picture of the fan operation itself. This assumption is very well confirmed both by general considerations and experimental results showing that the hydraulic power expenditure W_h for the same volume delivered is practically independent of the diffuser.

The computation method proposed is less simple than would be desirable. Its principle advantage, however, lies in the circumstances that it is free from such arbitrary assumptions and simplifications as may lead, in particular cases, to considerable error. The only exception is the condition assumed in the computation procedure; namely, that the entire width of the diffuser is momentarily filled out. In certain cases this gives a considerable deviation between the computation and test results.

IV. ILLUSTRATIVE EXAMPLE

As an example we shall compute the vaneless diffuser of an experimental high pressure fan, the fundamental geometric fan data being the following:

Outer impeller diameter $D_2 = 0.53$ meter

Diameter at diffuser entrance $D_3 = 0.33$ meter

Diameter at diffuser exit $D_4 = 0.65$ meter

Impeller width at exit $b_2 = 0.011$ meter

Diffuser width $b_3 = 0.011$ meter

The fundamental kinematic data are:

Volume Delivery = 300 cubic meters per hour

Tangential velocity at impeller exit $C_{2u} = 35.6$ meters per second

Radial velocity at impeller exit $C_{2m} = 7.3$ meters per second

The flow is assumed to fill out the entire exit cross section of the impeller, that is,

$$C_{2m} = \frac{Q_{sec}}{\pi D_2 b_2}$$

Bearing in mind that $D_2 = D_3$ and $b_2 = b_3$, we find that the radial velocity at the diffuser inlet $C_{3m} = C_{2m} = 7.3$ meters per second. The static head at the impeller outlet is $H_{st2} = 55.5$ millimeter of water. The friction coefficient λ is taken on 0.05.

The computation is most easily conducted by constructing the auxiliary curve $\alpha = f(R)$. To draw it it is sufficient to know the angle α_3 at the diffuser entrance and then to determine the angle α_a at the exit by making use of the equation

The flow inlet angle to the diffuser is

$$\alpha_a = \alpha_3 + \frac{\lambda \cos \alpha_3}{4b} (R_a - R_3)$$

The flow inlet angle to the diffuser is

$$\alpha_3 = \tan^{-1} \frac{C_{3m}}{C_{3u}} = \tan^{-1} \frac{7.3}{25.6} = \tan^{-1} 0.285$$

$$= 0.278 \text{ radians } (15^\circ 55')$$

$$\cos \alpha_3 \dots \dots \dots 0.962$$

$$\frac{\lambda \cos \alpha_3}{4b} \left(\frac{D_a - D_3}{2} \right) = \frac{0.05 \times 0.962}{4 \times 0.011} \left(\frac{0.65 - 0.33}{2} \right) = 0.175$$

$$\alpha_a = \alpha_3 + 0.175 = 0.278 + 0.175 = 0.453 \text{ radians } (26^\circ)$$

Knowing the values α_3 and α_a , we can construct the straight line $\alpha = f(R)$. (See fig. 5.) Then knowing the diffuser exit angle, we may determine also the absolute exit velocity C_a

$$C_a = \frac{C_{am}}{\sin \alpha_a}$$

where

$$C_{am} = \frac{Q_{sec}}{\pi D_a b}$$

The angle α increases in passing through the diffuser from $15^\circ 55'$ to 26° ; $\sin \alpha$ changes correspondingly from 0.275 to 0.439. For this case the application of the formula

$$\Delta H_r = K_\lambda K_{am} \rho \frac{C_{am}^2}{2} R_3^2 \left(\frac{1}{R_3} - \frac{1}{R_a} \right)$$

involves considerable inaccuracy. It is, therefore, necessary, as pointed out above, to subdivide the flow path into a number of intervals, construct the curve $dH_r/dR = f(R)$ and integrate graphically. The curve is constructed with the aid of the equation (15)

$$\frac{dH_r}{dR} = K_\lambda K_\alpha \rho \frac{C_m^2}{2} = \frac{\lambda \rho}{4b \sin \alpha} \frac{C_m^2}{2} \left(1 + \frac{1}{\tan^2 \alpha} \right)$$

Table I is the auxiliary table for the construction of the curve.

The curve (fig. 6) constructed from the above tabulated data very clearly brings out the fact that the losses are concentrated mainly at the initial part of the diffuser and that the increase in the losses starting from a certain diameter becomes very insignificant (at the same time there is a decrease in the static pressure rise). By measuring the area under the curve we find the magnitude of the friction loss.

The computation of the static pressure at the diffuser exit now evidently offers no difficulties and is carried out by the equation (all terms on the right side of which are known):

$$H_{st a} = H_{st 2} + \rho \frac{C_2^2}{2} - \Delta H_{tot} - \rho \frac{C_a^2}{2} = 55.5 + 43.2 - 18.2$$

$$-4.3 = 76.2 \text{ mm H}_2\text{O}$$

$$\left(\text{or } H_{sta} = H_{sta} + \rho \frac{C_{2m}^2}{2} + \rho \frac{C_{2m}^2}{2} - \Delta H_{tot} - \rho \frac{C_{am}^2}{2} - \rho \frac{C_{au}^2}{2} \right)$$

It should be noted that the equality of the velocities at the impeller outlet and diffuser inlet is a result of the filling out of the outlet width of the impeller and also of the equality of the widths of the impeller and diffuser for the given fan, a condition which by no means always obtains. In the general case, where there are impact losses, we have for ΔH_{tot} (where ΔH_{tot} = total loss in press).

$$\Delta H_{tot} = \Delta H_r + \Delta H_{imp}$$

In the given example, the computation appeared somewhat lengthy, not only because of the accompanying explanations, but also as a result of the considerable variation of $\sin \alpha$. In the majority of cases the computation is much simpler and shorter because of the possibility of applying formula (16), that is, computing with the mean values of $\sin \alpha$. The computation of the vaneless diffuser for three delivery volumes is presented in tables II, III, IV, V, and VI.

From the data of the last table the utilized portion of the blower characteristic was constructed (fig. 11). A comparison of this computed characteristic with the experimental curve shows good agreement between the two curves. For a value of λ lying within the range 0.05 - 0.15 such agreement is obtained also for the other tested blowers (fig. 12).

With regard to the correct estimate of the friction coefficient it should be remarked that for the generally encountered smooth metallic surfaces of fan diffusers the coefficient λ has a value half as large as for the roughly treated surfaces tested at CAHI (plywood surface diffusers). In accordance with this fact in computing diffusers with well worked over surfaces a value of λ of 0.03 - 0.07 should be used. The friction coefficient used by Pfleiderer also lies within this range.

Because of the good agreement of the experimental with the computed results the computation method presented above may be considered as sufficiently reliable.

V. VANELESS DIFFUSER AS AN ELEMENT OF THE CENTRIFUGAL FAN AND BLOWER

The vaneless diffuser has up to the present found comparatively small application to fans. Such, for example, are the fans of the Rateau type (fig. 13) and the fans of medium and high pressure of the CAHI type with vaneless diffuser (figs. 14 and 15). In the case of multistage turbo blowers and turbo compressors, however, the vaneless diffuser has long since found application (fig. 16). In the majority of cases of multistage design, however, bladed diffusers are used (fig. 17). In some particular designs these two diffuser types are combined, a vaneless diffuser being used for some of the stages and guide vanes for others. An example of such a design is the three-stage blower shown in figure 18.

Without going into a detailed relative comparison analysis of the performance of the vaneless with the bladed diffuser, we may note merely that the high pressures and efficiencies of the latter are observed for a very much smaller range of outputs than are observed for the former (fig. 19). This appears as the result of eddy formation arising at the initial part of the diffuser at outputs other than rated when the flow strikes the blades. The magnitude of this eddy formation and the losses associated with it, to a large degree, depend on the angle at which the flow attacks the blades. In those cases where the range of outputs utilized is large, the application of vanes is necessarily associated with large angles of attack and hence with large turbulence losses at outputs considerably larger than the one for which the blower was computed. This leads to a considerable lowering of the pressure and efficiency at the corresponding deliveries.

The advantage of these diffusers, however, lies in the possibility of an appreciable lowering in the velocity without a large increase in the dimensions of the machine and also in somewhat higher efficiencies at rated delivery volume. If a large diffuser size (large value of D_a/D_3) is not suitable for reasons of space saving the advantage will lie with the bladed diffuser. In the great majority of cases, the final selection is decided upon after making a comparison computation. For preliminary rough computations we may assume that for angles of attack $\phi < 12^\circ$ the bladed diffuser is preferable and for $\phi > 19^\circ$ the vaneless diffuser. For fans

the bladed diffuser is not applied since the construction would thereby become considerably complicated.

In concluding we may note that, besides the vaneless diffusers of constant width, diffusers with increasing width are also encountered. The computation of diffusers of such type is not considered in our present paper.

VI. OTHER COMPUTATION METHODS*

Following upon the work of Pfleiderer (reference 2) on centrifugal pumps, a work which for several years has been of fundamental aid to engineers of other countries as well as of Germany, the problem of the vaneless diffuser has been fully and seriously investigated. As has already been mentioned our computation scheme here presented is based on the work of Pfleiderer (reference 3). The latter considers the problem from the most general point of view: namely, the operation of a vaneless diffuser of variable width. The mathematical complexity of the expressions obtained makes it necessary for him to neglect the radial velocity component C_m in comparison with the tangential C_u .** For particular fans and certain operating conditions (small C_m/C_u) this simplification is entirely admissible. In all other cases, however, when the value of the radial and tangential velocities are of the same order of magnitude the simplification is not justified. These considerations hold not only for vaneless diffusers of constant width, but also for diffusers of increasing width (for which the radial component decreases more rapidly) because in all cases the losses are essentially concentrated at the initial part of the diffuser.

In the case of vaneless diffusers of constant width, Pfleiderer dispenses with the above-mentioned assumption and then obtains an equation for the determination of the flow exit angle in the general form and an equation for the magnitude of the losses for the particular case $\alpha_3 = 90^\circ$. The expression obtained for the determination of the angle α_a is considered by him as inapplicable to practical cases.

*Pfleiderer, Kearton, Ostertag.

** The obtained expression for the magnitude of the losses was nevertheless so complicated that it could be used only with the aid of supplementary graphical constructions.

The further treatment and simplification in our present paper of the expressions obtained by Pfleiderer lead to practically applicable formulas and to a computation method. The corresponding tests and investigations have made it possible to define more accurately the range of values of the friction coefficient and have given a practically sufficient confirmation of the validity of the method for computing the actual flow process. Our present paper is to be considered as a development of a number of ideas set forth by Pfleiderer as early as 1927.

In his book "Turbo-blowers and Compressors" Kearton presents a computation scheme which only broadly corresponds to the physical nature of the process investigated since he admits two fundamental errors. The first serious error consists in the fact that he considers the trajectory of the real flow in the vaneless diffuser to be a logarithmic spiral. It has been shown above that under the influence of friction the values of the tangential components are less than would be the case in accordance with the law of constancy of circulation; the spiral unrolling at a more rapid rate. This fact is significant not only in that it affects the magnitude of the losses but also in that it affects the magnitude of the flow exit angle, which is increased (C_u decreases for constant C_m). In the case of turbocompressors or turboblowers with backward curved guide vanes this error may have as a consequence that instead of the expected shockless entry at the backward curved guide vanes, or entry with very small positive angle of attack, the flow will actually enter the guide vanes with a negative angle of attack. The question as to precisely at what angles of attack, positive or negative, the losses are less, can not as yet be looked upon as answered. There is no doubt, however, that certain phenomena that are associated with negative angles of attack render the corresponding computation of little reliability.

The second error consists in the fact that the method of computation, using the average hydraulic radii and velocities at inlet and outlet, while admissible in particular cases, is recommended by Kearton as a general computation method. He writes "A simpler and, at the same time, sufficiently accurate method is that of computing the mean specific volume v_m , mean area F_m , total surface S and mean resistance R , and the energy loss is then equal to $R S v_m / F_m \text{ kgm}$ ". As a matter of fact, the losses are concentrated mainly at the entrance to the diffuser. At the exit, however, they are relatively small and starting with a certain diameter any further increase has practically no

effect on the magnitude of the losses. For these reasons the losses computed according to the mean values of the velocity and hydraulic radius are not characteristic magnitudes and may lead to errors of the order of 50 percent and over. For the same reasons it would have been more correct to have substituted instead of $F_m = (F_{inlet} + F_{outlet})/2$ the magnitude $(1/F_m) = (1/F_{inlet} + 1/F_{outlet})/2$.

In the classical treatise of Professor Ostertag (reference 4) only an approximate computation scheme of vaneless diffusers is presented; while certain remarks of the author are quite incorrect. The fundamental equation for determining the pressure rise in the vaneless diffuser is, according to Ostertag, of the following form:

$$h_d = \frac{1}{2g} \left[(1 - \xi) C_2^2 - \left(C_{2u} \frac{R_2}{R_a} \right)^2 - C_{am}^2 \right]$$

The physical significance of the above equation is perfectly clear: namely; that the pressure rise in the vaneless diffuser is equal to the difference in the dynamic pressures at the diffuser inlet and outlet, less the losses relative to the velocity at the impeller outlet. The tangential velocity component at the diffuser outlet is determined by the law $RC_u = \text{constant}$. In correspondence with these equations Ostertag shows that the flow in a vaneless diffuser moves along a logarithmic spiral. He thus, like Kearton, ignores the decrease in the tangential velocity due to friction.* We have already discussed the consequences of this assumption.

The determination of the losses with the aid of the coefficient ξ referred to the dynamic pressure at the impeller outlet, cannot be considered successful since it essentially presupposes nondependence of the losses on the diffuser size. In noting the effect of friction on the magnitude of the velocity Ostertag observes that the friction decreases the tangential component to a greater degree than the radial. Actually the friction decreases only the tangential component since for a given discharge volume, Q_m^3/sec , the radial component is independent of the friction being equal to $Q_{sec}/\pi Db$.

*The work of Ostertag was published considerably earlier than that of Kearton.

Comparing the work of Ostertag with that of Kearton it may be said that the former investigated the performance of the vaneless diffuser more thoroughly than Kearton, who considered the problem only in its broadest features. This is shown partly by the fact that Ostertag associates the coefficient of friction in the diffuser ζ with the initial velocity; whereas, Kearton recommends computing by the mean velocities. The estimate of the losses in the diffuser with the aid of the loss coefficient ζ in the absence of reliable data for the choice of values of this coefficient may, however, lead to results that are in sharp disagreement with the true values.

VII. APPLICATION OF THE VANELESS DIFFUSER TO AXIAL FANS

The desirability of applying the vaneless diffuser to axial fans follows from the consideration that the dynamic pressure corresponding to the tangential component produced by the axial fan can not be directly utilized and the energy expended in producing the tangential velocity component when the air is discharged to the atmosphere or to a pressure line thus appears as lost energy.

In emphasizing the negative role (in this case) of large tangential velocities we, by no means, wish to say that these components must be reduced to zero (it should not be forgotten then that the total theoretical head is $H_{Th} = \rho U_2 C_{2u}$). The fact is that the corresponding energy loss differs in principle from the hydraulic losses in that the former can not be considered as irreversibly lost. It is possible to utilize almost entirely the quantity $\rho C_{2u}^2/2$ by transforming it into pressure. It is therefore desirable that the performance of the casing in raising the pressure be effective so as to permit large values of C_{2u} .

To transform velocity into pressure the fan is provided with guide vanes (fig. 20) or is enclosed in a spiral casing (fig. 21) or, finally, is provided with a vaneless diffuser. The construction scheme of a vaneless diffuser is clear from the drawing (fig. 22). To avoid additional losses in turbulence, a guide element is located directly behind the hub: namely, a short cylindrical pipe of the same diameter as the hub and widening out at the diffuser.

In considering the performance of the vaneless diffuser we shall, in order to simplify the computation, substitute

for the actual process a provisional one in which there is no rounding of the inner edge for the purpose of decreasing the eddy formation in rotating. We shall say a few words on the effect of the distance of the diffuser from the fan or more accurately, from the frame in which the fan is mounted (this distance is equal in magnitude to the width of the diffuser).

Experiments and computations show that on increasing the dimension b , (width of casing) within the limits of those values for which $C_{3m} > C_{2a}$ the effect of the casing is to increase H_{sta} and η_{st} . On further increasing the dimension b for which $C_{3m} < C_{2a}$ experiment shows a reverse effect, namely, a decrease of H_{sta} and η_{st} . The reason for this is the separation of the flow from the walls of the casing at values of b greater than a certain critical value, for which the computation ceases to be valid.

The above considerations with regard to the optimum width of the casing must be considered as approximate only since no special series of tests on fans with a wide range of widths has been undertaken. The optimum value of b on the basis of these considerations is determined by the equation

$$C_{3m} = C_{2a}$$

or

$$\pi D_2 b \mu = \frac{\pi}{4} (D_2^2 - d^2) = \frac{\pi D_2^2}{4} (1 - \xi^2)$$

whence

$$b = \frac{0.25}{\mu} D_2 (1 - \xi^2) \approx 0.25 D_2$$

The computation is conducted on the basis of the characteristic experimentally obtained on the fan operating without casing. The analysis of the process explained above may be almost entirely applied to the axial fan. It is only necessary to consider further the process occurring between the fan and the diffuser inlet.

At the fan outlet the magnitude and direction of the absolute velocity is determined by the two components, the axial and the tangential. The axial velocity is

$$C_{2a} = \frac{Q_{sec}}{F} = \frac{Q_{sec}}{\pi/4(D_2^2 - d^2)}$$

where d is the diameter of the hub. The tangential component at the diffuser inlet, that is, at the diameter D_3 , is determined from the power with the aid of the formula given at the beginning of this paper.*

To represent fully the state of the flow at the diffuser inlet the following factors must be taken into account:

1. The flow from the fan to the diffuser is necessarily accompanied by hydraulic rotational losses.
2. A greater or less compression of the flow with subsequent expansion at the diffuser inlet may be expected.
3. A result of this expansion is hydraulic impact.

We shall consider these three points in greater detail.

1. The rotational losses are evaluated in terms of the dynamic pressure corresponding to the velocity C_{3m} . The magnitude of the corresponding loss coefficient ξ in the presence of rounding may be taken equal approximately to 0.2. In the absence of rounding, the magnitude of the loss coefficient may be raised to 1.5.

The problem of the choice of the coefficient ξ may be solved on the basis of the same considerations which

*For volumes approaching zero the above method ceases to be valid. This is due to the fact that for very small volumes the computed tangential velocity evidently tends to infinity since C_u is inversely proportional to the volume ($C_{2u} = 75 W_h / Q_p U_2$). Actually the fan then operates as a mixer and the velocity C_{2u} approaches the value of the peripheral velocity, the power being dissipated in eddy formation in the associated masses. The computed characteristic of the fan for very small discharges is practically of no value.

determined the value of this coefficient in the case of the usual blower*

Thus

$$\Delta H_{rot} = \xi \rho \frac{C_{3m}^2}{2}$$

(More will be said below on the value of C_{3m} .)

2. The compression of the flow may with sufficient accuracy be taken into account with the aid of the test curve shown in figure 23 in which the compression coefficient μ , the ratio of the width of the active flow (at the diameter D_3) to the total width of the diffuser; that is,

$$\mu = \frac{b_{flow}}{b}$$

is plotted against the ratio b/D_3 . The radial velocity of the flow at the diffuser inlet is therefore

$$C_{3m} = \frac{Q_{sec}}{\pi D_3 b_{flow}} = \frac{Q_{sec}}{\pi D_3 b \mu}$$

3. As in the case of the centrifugal impeller we assume, in order to simplify the actual process somewhat that the radial velocity at the edge of the diffuser inlet, C'_{3m} , is equal to

$$\frac{Q_{sec}}{\pi D_3 b}$$

where

$$D_3 = D_2$$

*We restrict ourselves in the present paper to pointing out the limiting values of ξ , referring the reader who desires fuller information to the article by G. Abramovich (reference 5).

The impact losses are

$$\Delta H_{\text{imp}} = \frac{\rho}{2} (C_{2a} - C'_{3m})^2$$

or

$$\Delta H_{\text{imp}} = \frac{\rho}{2} (C_{3m} - C'_{3m})^2$$

if

$$C_{3m} > C_{2a}$$

The above method assumes, in particular, that the presence of the vaneless diffuser does not change either the kinematics of the flow through the fan or the pressure behind the fan. In the case of centrifugal fans this assumption is sufficiently well founded. In the case of axial fans it somewhat contradicts the principle of the phenomenon itself. In the presence of a diffuser the pressure at the axis should drop and the discharge volume correspondingly rise in comparison with the case of operation without diffuser. At the same time experiment shows that the presence or absence of a diffuser has little effect on the power expenditure. This fact justifies the application of the above assumption not only for centrifugal but also for axial fans.

We present below the computation of an axial fan of the CAHI type; figure 24 shows the computed and test curves of the static head H_{sta} . (See table VII.)

CONCLUSION

The computation scheme of Professor Pfleiderer is developed and worked over so as to provide a practical method for the computation of vaneless diffusers. The method is simultaneously free from such assumptions and simplifications as impair the true physical picture of the process or are applicable only in particular cases. Comparison of the computations with experimental data shows sufficiently good agreement and permits the determination of the limits within which the friction coefficient λ lies. The conclusions

that were found applicable to the centrifugal fan are then generalized for axial fans.

Translation by S. Reiss,
National Advisory Committee
for Aeronautics.

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TABLE I

AUXILIARY TABLE FOR CONSTRUCTION OF CURVES OF FIGURES 5 AND 6

Number	D	R-R ₃	α°	$\sin \alpha$	$\tan \alpha$	C_m	K_{α}^*	K^{**}	$\frac{dH}{dR}$
1	330	0.00	15° 55'	0.274	0.285	7.3	48.6	6.75	360
2	350	.01	16° 50'	.284	.296	6.88	43.7	6.09	288
3	370	.02	17° 12'	.296	.309	6.50	38.6	5.36	226
4	410	.04	18° 27'	.316	.333	5.87	31.6	4.39	151
5	470	.07	20° 18'	.346	.369	5.12	24.1	3.35	88
6	550	.11	22° 50'	.388	.420	4.38	17.2	2.39	46
7	650	.16	26° ----	.438	.487	3.70	11.9	1.65	23

 $\Delta H_r = 18.15$ mm water

$$*K_{\alpha} = \frac{1}{\sin \alpha} \left(1 + \frac{1}{\tan^2 \alpha} \right)$$

$$**K = K_{\alpha} K_{\lambda} \rho/2$$

$$\text{where } K_{\lambda} = \frac{\lambda}{2b} \approx 2.28$$

Table II for computing angle α and flow velocity in vaneless diffuser

Q_{HR}	M^3/HR	300	500	700
Q_{sec}	M^3/sec	0.0834	0.139	0.1945
$C_{3m} = \frac{Q_{sec}}{\pi D_3 b_3} = C_{2m}$	M/sec	7.3	12.2	17
$C_{3u} = C_{2u}$	"	25.6	23.4	20.3
$C_3 = \sqrt{C_{3m}^2 + C_{3u}^2}$	"	26.6	26.4	26.5
$\alpha_3 = \arctan \frac{C_{3m}}{C_{3u}}$	"	15°-55'	27°-40'	39°-50'
$\cos \alpha_3$		0.961	0.885	0.765
$\frac{\lambda}{4b}(R_a - R_3) = \frac{0.05}{4 \times 0.011} \cdot 0.16 = 0.182$		-	-	-
$\alpha - \alpha_3 = \frac{\lambda}{4b}(R_a - R_3) \cdot \cos \alpha$	Radians	0.175	0.165	0.140
$\alpha - \alpha_3$		10°-3'	9°-20'	8°-01'
$\alpha_a = \alpha_3 + (\alpha - \alpha_3)$		26°-0'	37°	47°-50'

Auxiliary tables for computing curve $\frac{dH}{dR} = f(R)$
for delivery volumes of 300, 500 & 700 M^3/HR .

Table III $Q = 360 M^3/HR$

Nº	D	$R - R_3$	α°	$\sin \alpha$	$\tan \alpha$	C_m	K_α	K	$\frac{\Delta H}{\Delta R}$
1	330	0	15°-55'	0.274	0.285	7.3	48.6	6.75	360
2	350	0.01	16°-30'	0.284	0.290	6.88	43.7	6.09	288
3	370	0.02	17°-12'	0.296	0.309	6.50	38.6	5.36	226
4	410	0.04	18°-27'	0.316	0.333	5.87	31.6	4.39	151
5	470	0.07	20°-18'	0.346	0.369	5.12	24.1	3.35	88
6	550	0.11	22°-50'	0.388	0.420	4.38	17.2	2.39	46
7	650	0.16	26°	0.438	0.487	3.70	11.9	1.65	32

Table IV $Q = 500 \text{ M}^3/\text{Hr}$

Nº	D_{MM}	$R-R_{3M}$	α°	$\sin \alpha$	$\tan \alpha$	C_m	K_α	K	$\frac{\Delta H}{\Delta R}$
1	330	0	27°40'	0.465	0.524	12.2	10.1	1.41	210
2	350	0.01	28°18'	0.474	0.538	11.5	9.68	1.345	178
3	370	0.02	28°50'	0.482	0.551	10.9	8.92	1.24	148
4	410	0.04	30°	0.50	0.577	9.8	7.0	1.11	107
5	470	0.07	31°42'	0.525	0.618	8.6	6.91	0.96	71
6	550	0.11	34°06'	0.561	0.677	7.3	5.7	0.792	42.1
7	650	0.16	37°	0.602	0.754	6.2	4.59	0.638	24.5

Table V $Q = 700 \text{ M}^3/\text{Hr}$

Nº	D_{MM}	$R-R_{3M}$	α°	$\sin \alpha$	$\tan \alpha$	C_m	K_α	K	$\frac{\Delta H}{\Delta R}$
1	330	0	39°50'	0.641	0.834	17.00	3.79	0.526	152
2	350	0.01	40°21'	0.646	0.850	16.00	3.69	0.513	132
3	370	0.02	40°50'	0.654	0.864	15.15	3.57	0.496	114
4	410	0.04	41°48'	0.666	0.894	13.70	3.38	0.470	88.2
5	470	0.07	43°20'	0.686	0.944	11.92	3.10	0.430	61.1
6	550	0.11	45°18'	0.711	1.012	10.2	2.76	0.384	40.0
7	650	0.16	47°51'	0.741	1.105	8.64	2.45	0.341	25.5

Table VI for computing static pressure at vaneless diffuser exit.

Q_{HR}	M^3/HR	300	500	700
ΔH_r - - - - -	MM H ₂ O	18.15	13.21	10.70
C_L - - - - -	M/sec	26.6	26.4	26.5
$e \frac{C_2^2}{2}$ - - - - -	MM H ₂ O	43.2	42.5	42.9
C_a - - - - -	M/sec	8.45	10.3	10.55
$e \frac{C_a^2}{2}$ - - - - -	MM H ₂ O	4.35	6.5	8.3
$\Delta H_{imp} = 0 (C_{2m} = C_{3m})$ - - - - -		-	-	-
(H_{st2}) - - - - -	MM H ₂ O	55.5	41.5	24.50
$H_{sta} = H_{st2} + e \frac{C_2^2}{2} - \Delta H_r - e \frac{C_a^2}{2}$	"	76.2	64.3	48.4

Computation of vaneless diffuser for axial fan of
CAH-I type ($\lambda = 0.05$)

$b = 0.10 \text{ M}$
 $D_2 - D_3 = 0.710 \text{ M}$
 $D_4 = 1.8 \text{ M}$
 $U_2 = 74.5 \text{ M/sec.}$
 $\mu = 0.83$
 $\eta = 2,000 \text{ rpm}$
 $F_{\text{suopt}} = 0.241 \text{ M}^2$

Table VII

$Q \text{ M}^3/\text{hr}$ — — — — —	4000	6000	8000
$Q \text{ sec}$ — — — — —	1.11	1.67	2.22
$C_{2a} = \frac{Q_{\text{sec}}}{F_{\text{suopt}}}$ — — — — —	4.6	6.93	9.20
$C_{3m} = \frac{Q_{\text{sec}}}{\pi D_3 b_3}$ — — — — —	4.98	7.48	9.95
$C_{2u} =$ — — — — —	12.86	10.86	11.30
$C_2 = \sqrt{C_{2u}^2 + C_{2a}^2}$ — — — — —	13	11.2	11.9
$\sin \alpha_3 = C_{3m}/C_3$ — — — — —	0.383	0.667	0.835
α_3 — — — — —	22°-30'	41°-50'	56°-30'
$\cos \alpha_3$ — — — — —	0.925	0.745	0.55
$\frac{\lambda}{4b} (R_a - R_3) = \frac{0.05}{4 \times 0.10} (0.90 - 0.355) = 0.063$	—	—	—
$\alpha_a - \alpha_3 = \frac{\lambda}{4b} (R_a - R_3) \cos \alpha_3$ — (radians) —	0.0629	0.0506	0.0374
$\alpha_a - \alpha_3$ — — — — — (°) —	3°-36'	2°-54'	2°-08'
$\alpha_a = \alpha_3 + (\alpha_a - \alpha_3)$ — — — — —	26°-06'	44°-44'	58°-38'
$\sin \alpha_a$ — — — — —	0.44	0.704	0.854
$\sin \alpha_{cp} \text{ (mean)}$ — — — — —	0.411	0.686	0.844
$\tan \alpha_3$ — — — — —	0.172	0.80	2.30
$\tan \alpha_a$ — — — — —	0.240	0.98	2.70
$(\tan \alpha)_{cp} \text{ (mean)}$ — — — — —	0.206	0.89	2.50
$\frac{\lambda}{2b} = 0.25$	—	—	—
$K = \frac{\lambda b}{4b \sin \alpha_{cp}} (1 + \frac{1}{\tan^2 \alpha_{cp}})$ — — — — —	0.216	0.0473	0.025
$R_3^2 (\frac{1}{R_3} - \frac{1}{R_a}) = 0.216$	—	—	—
$K C_{3m} \cdot 0.216 = H_{fr}$ — — — — —	1.16	0.57	0.537
$Cam = \frac{Q_{\text{sec}}}{\pi D_4 b} = \frac{Q_{\text{sec}}}{0.565}$ — — — — —	1.97	2.96	3.93
$C_{4u} = \frac{Cam}{\tan \alpha_a}$ — — — — —	4.02	2.99	2.40
$C_a = \frac{Cam}{\sin \alpha_a}$ — — — — —	4.47	4.21	4.6
$\rho \frac{C_a^2}{2}$ — — — — —	10.1	7.2	7.8
$\rho \frac{C_a^2}{2}$ — — — — —	1.22	1.09	1.29
$H_{st \text{ exp.}}$ — — — — —	39	29	19
$C_{3m} = \frac{Q_{\text{sec}}}{\pi D_3 b \mu} = \frac{Q_{\text{sec}}}{0.185}$ — — — — —	6.0	9.0	12.0
$\Delta H_{imp.} = e/2 (C_{3m} - C'_{3m})^2$ — — — — —	0	0.1	0.3
$H_{st \text{ dif}} = H_{st \text{ exp.}} + e \frac{C_a^2}{2} - \Delta H_{fr} - \Delta H_{imp} - \rho \frac{C_a^2}{2}$	46.72	34.44	24.68

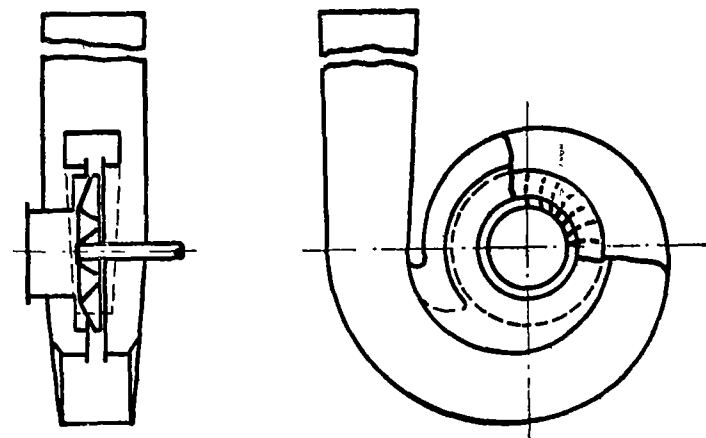
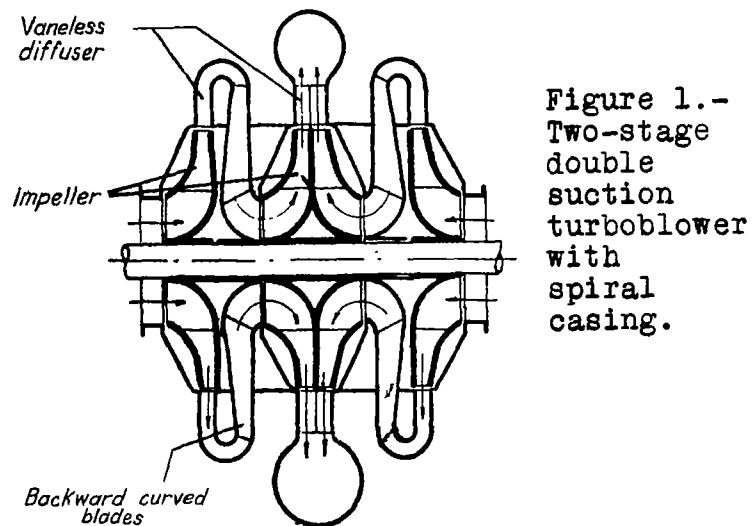


Figure 13.- Sketch of a centrifugal mine fan of the Rateau type.

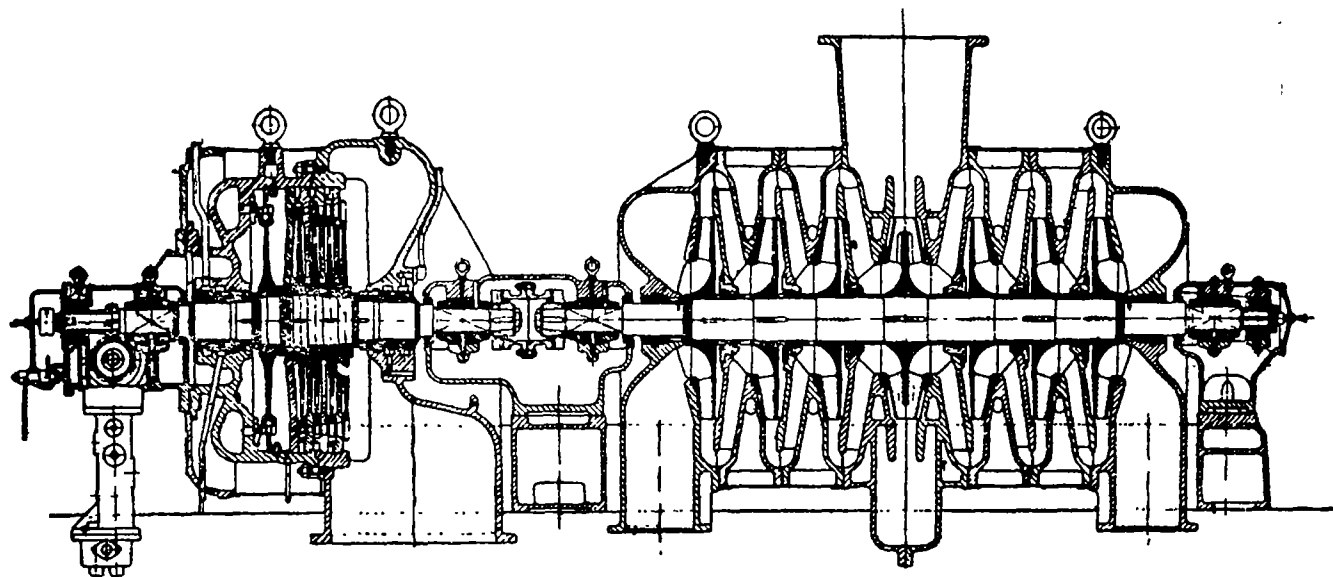


Figure 16.

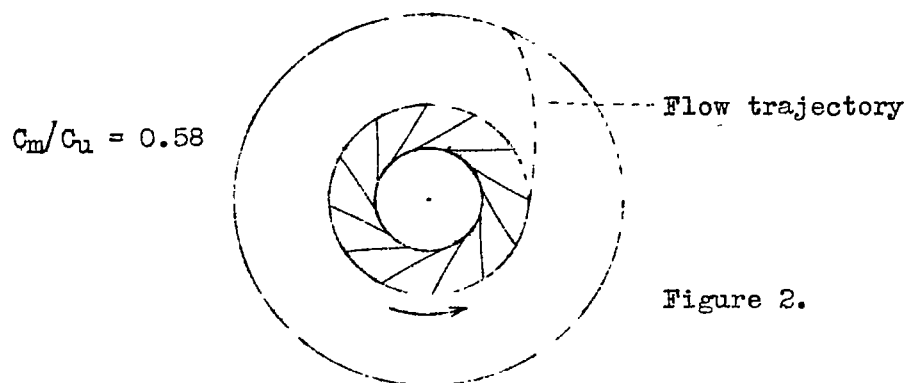


Figure 2.

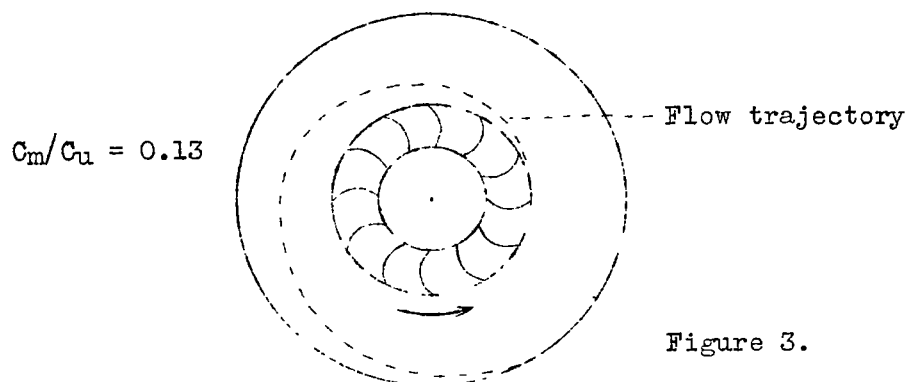


Figure 3.

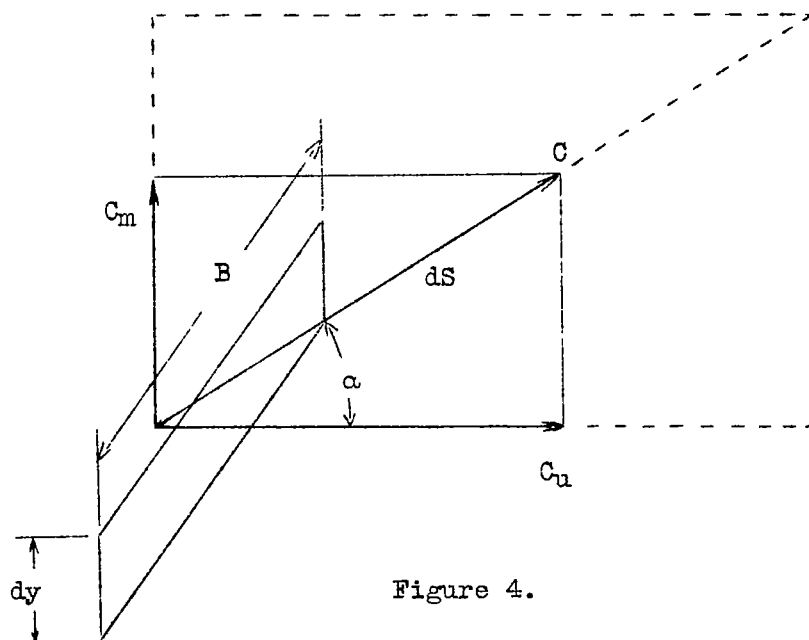


Figure 4.

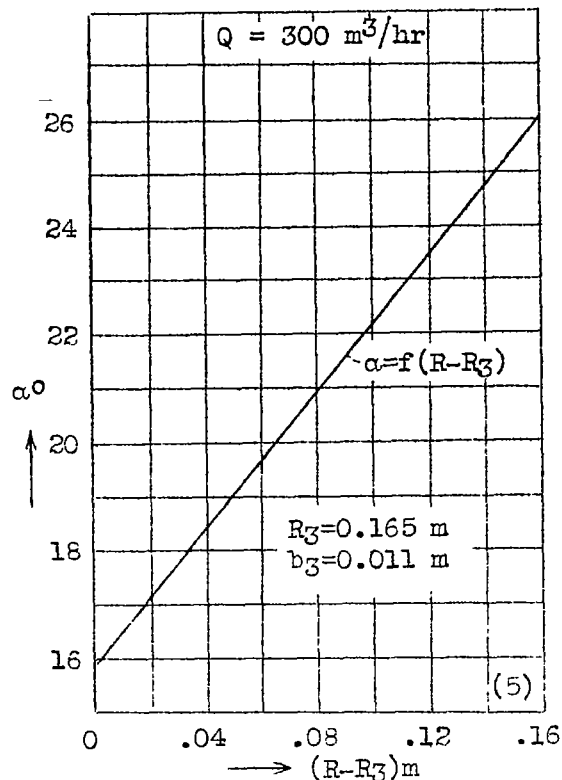


Figure 5.- Change in the angle α in flowing through the vaneless diffuser, $Q = 300 \text{ m}^3/\text{hr}$.

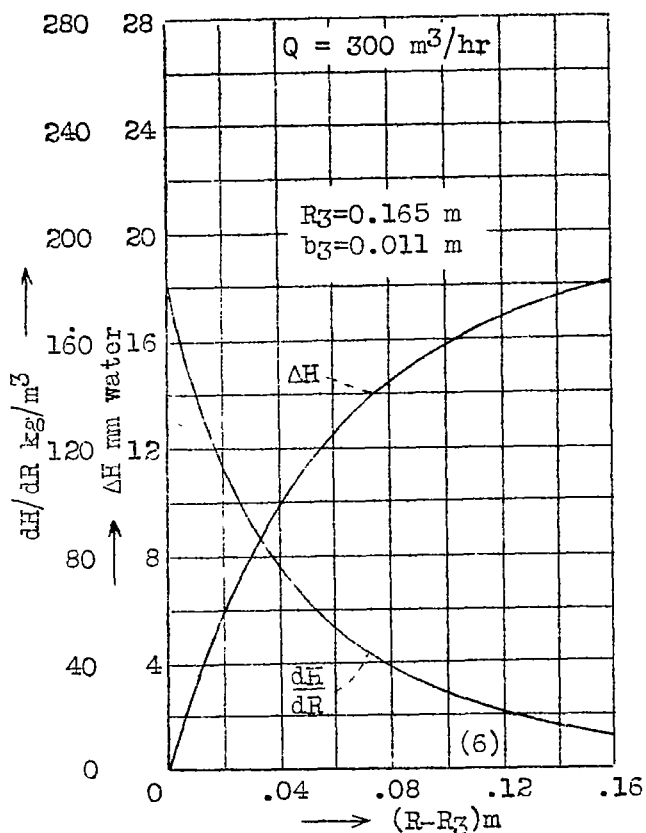
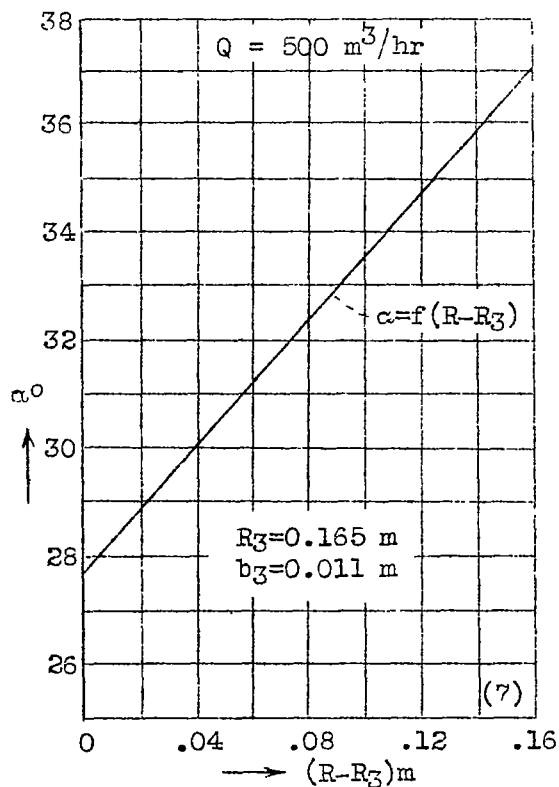


Figure 6.- Increase in the losses in flowing through the vaneless diffuser, $Q = 300 \text{ m}^3/\text{hr}$.

Figure 7.- Change in the angle α in flowing through the vaneless diffuser, $Q = 500 \text{ m}^3/\text{hr}$.

Figure 9.- Increase in the losses in flowing through the vaneless diffuser, $Q = 500 \text{ m}^3/\text{hr}$.

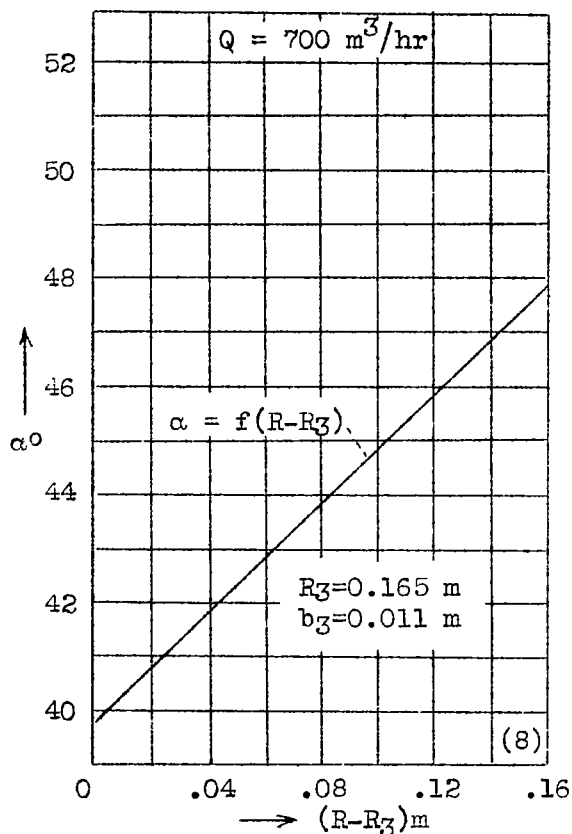
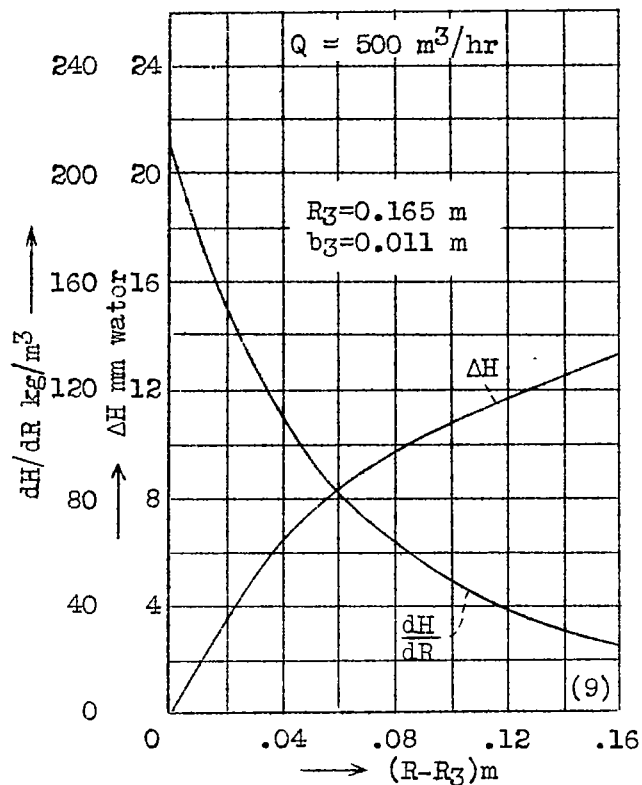
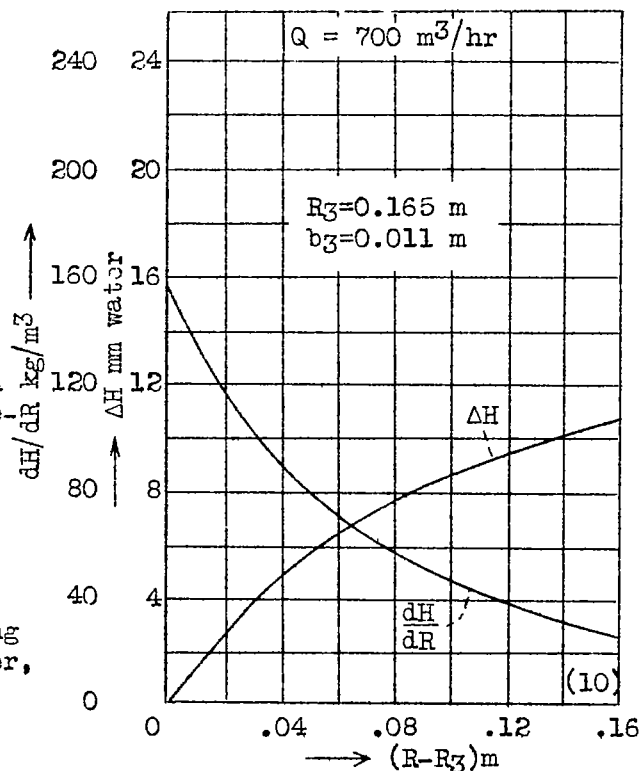


Figure 8.- Change in the angle α in flowing through the vaneless diffuser, $Q = 700 \text{ m}^3/\text{hr}$.

Figure 10.- Increase in the losses in flowing through the vaneless diffuser, $Q = 700 \text{ m}^3/\text{hr}$.



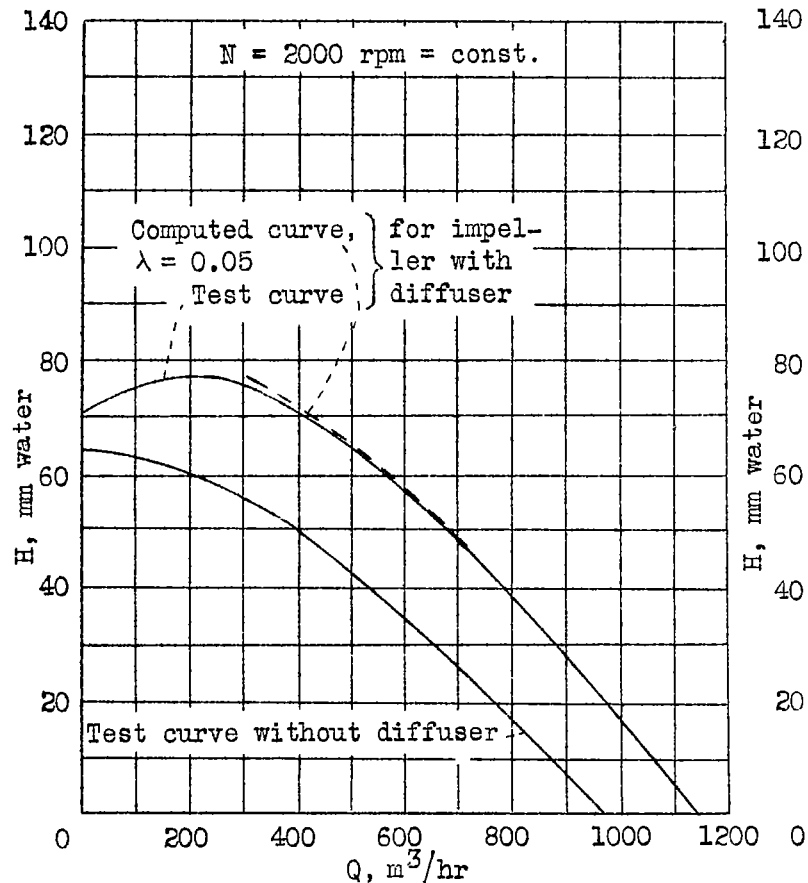


Figure 11.- Computed and test curves for H_{sta} for a high pressure fan.

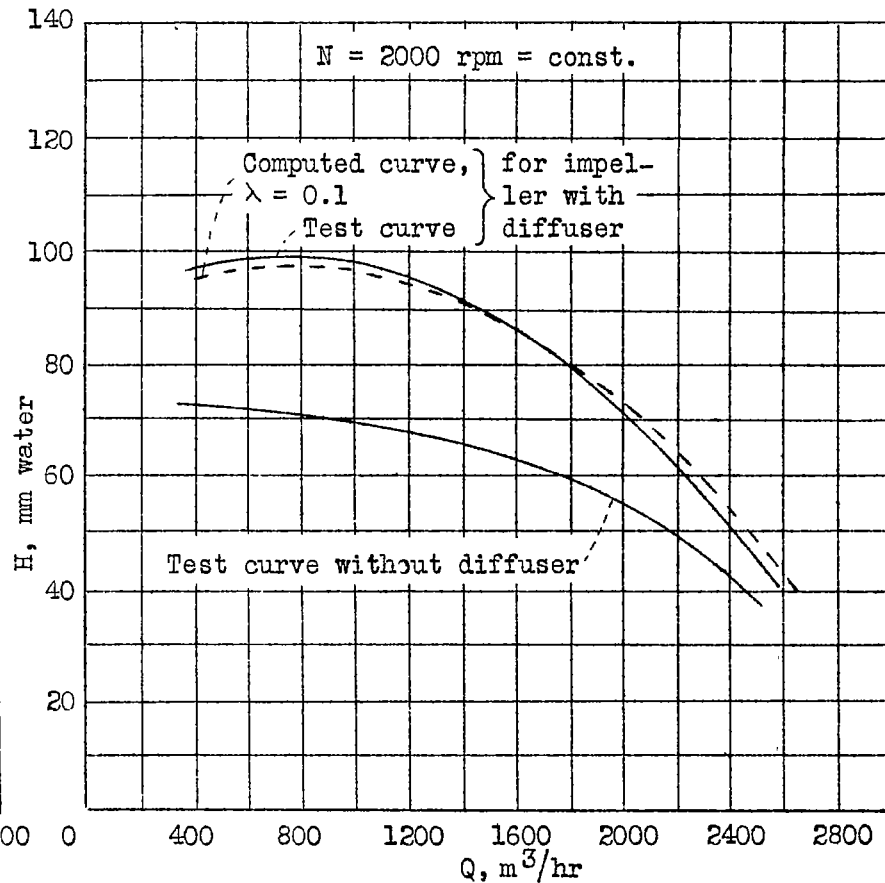


Figure 12.- Computed and test curves for H_{sta} for the first stage of a model of a two-stage fan.

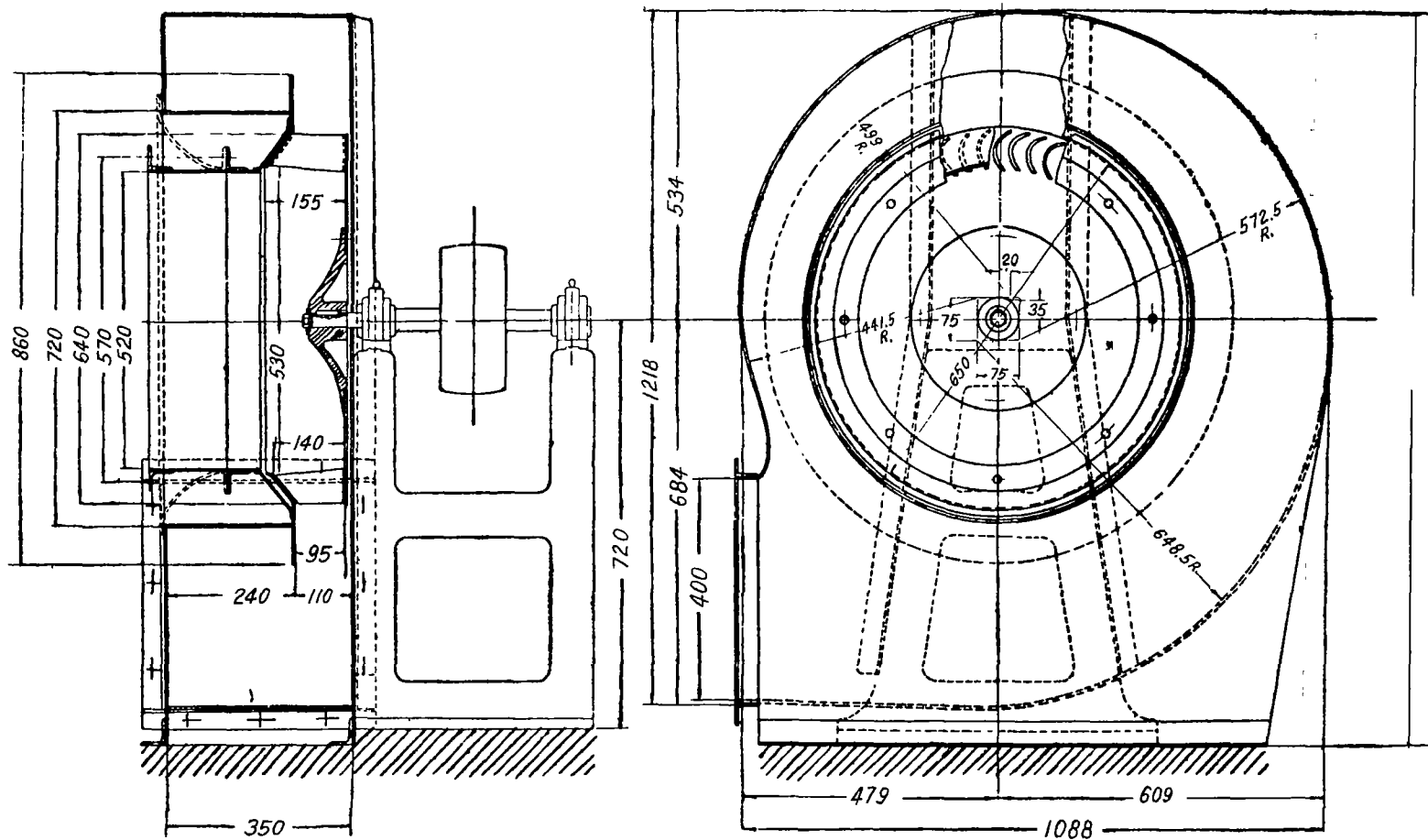


Figure 14.- Sketch of a centrifugal fan of medium pressure of the CAHI type.

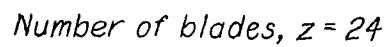


Figure 15.- Centrifugal high pressure fan of the CAHI type.

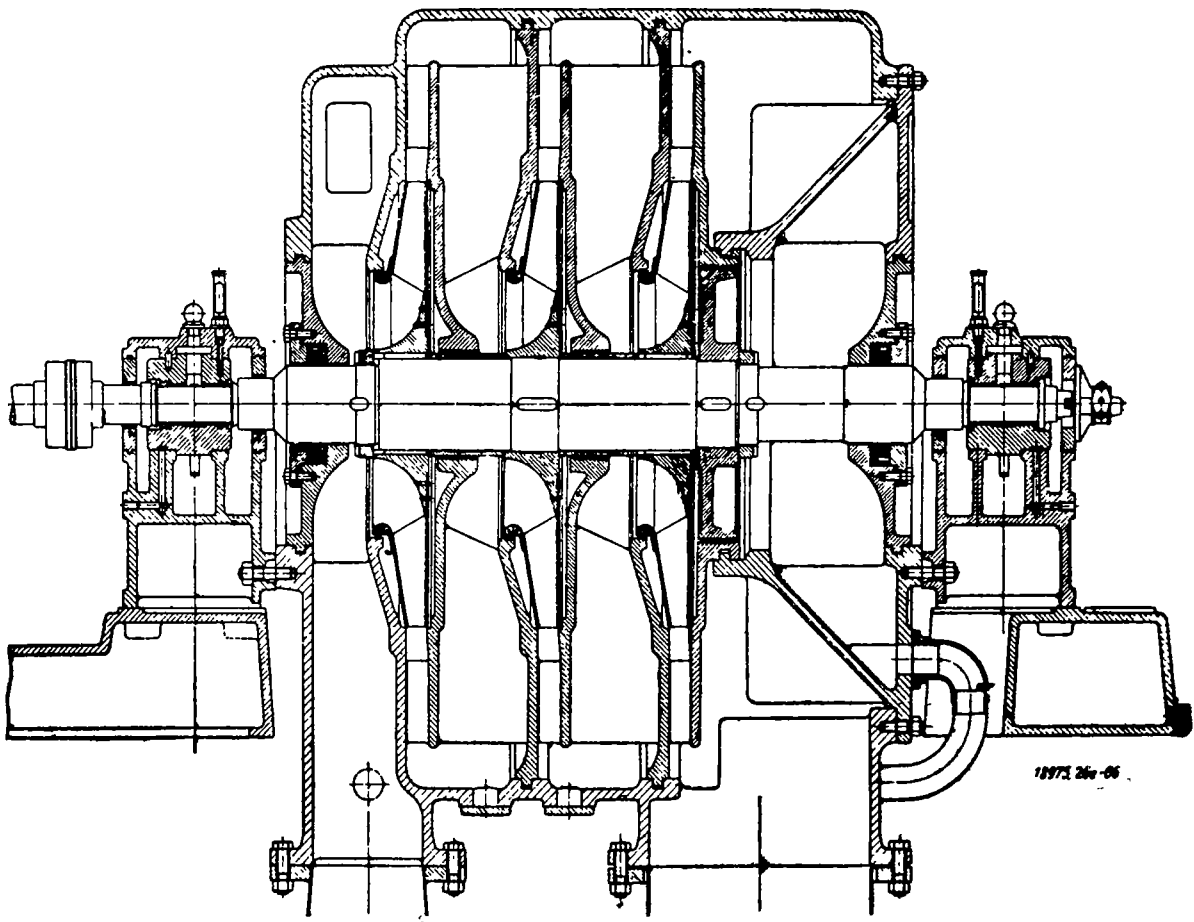
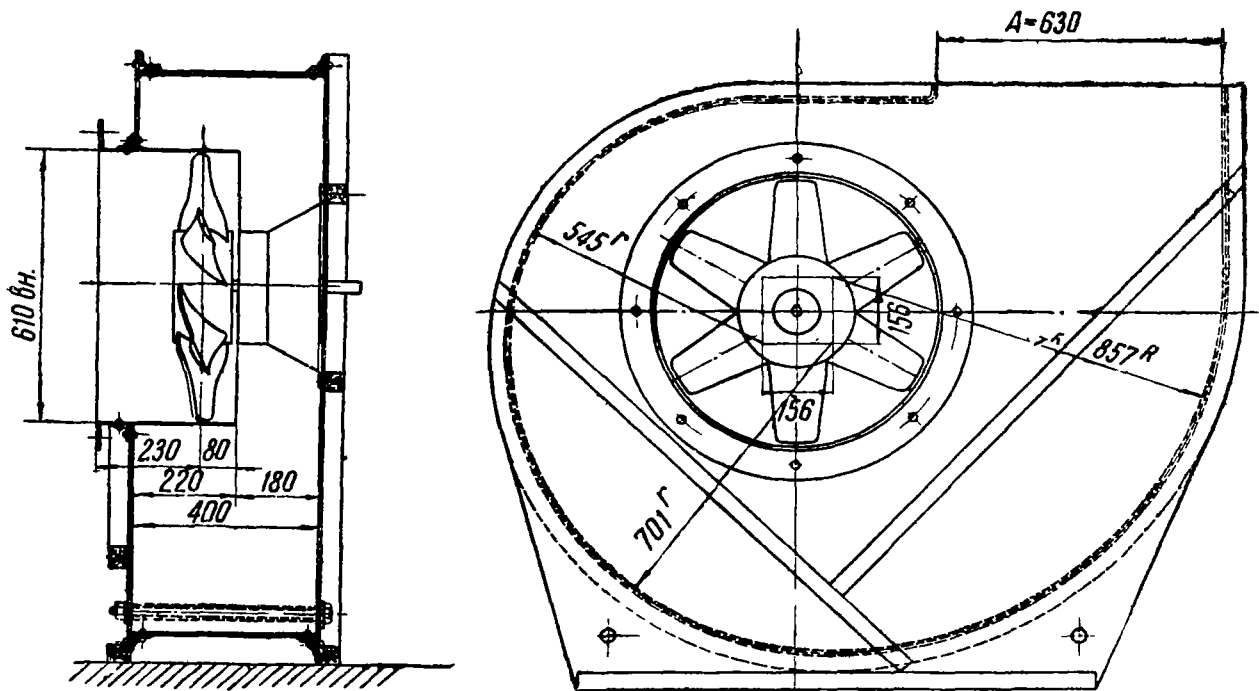
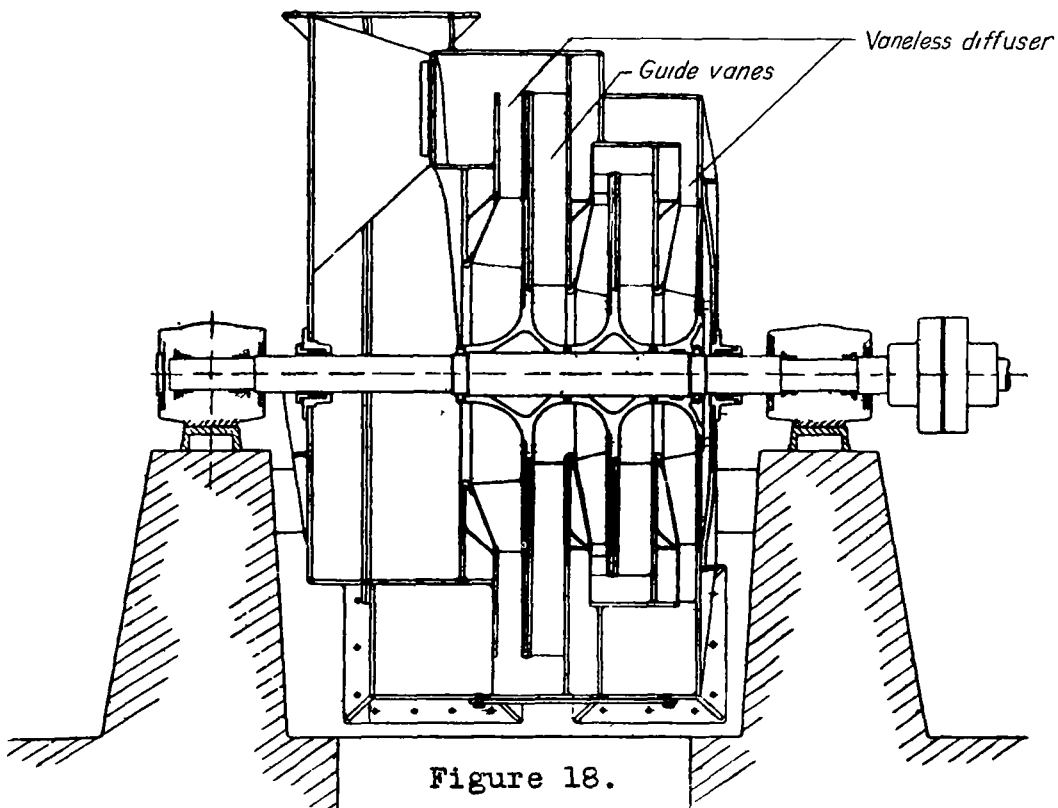


Figure 17.



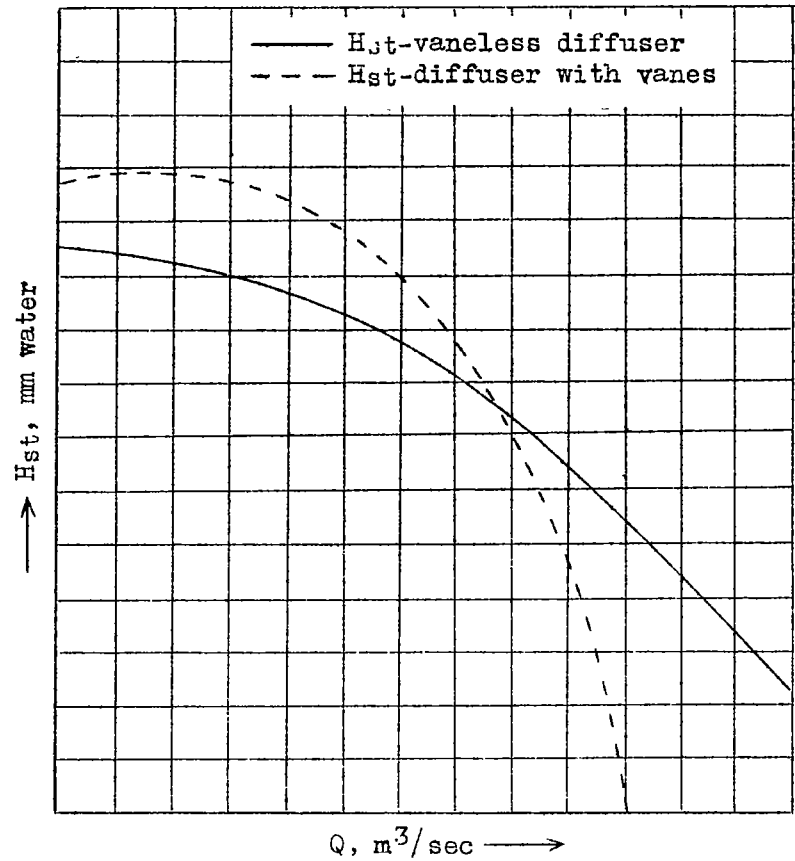
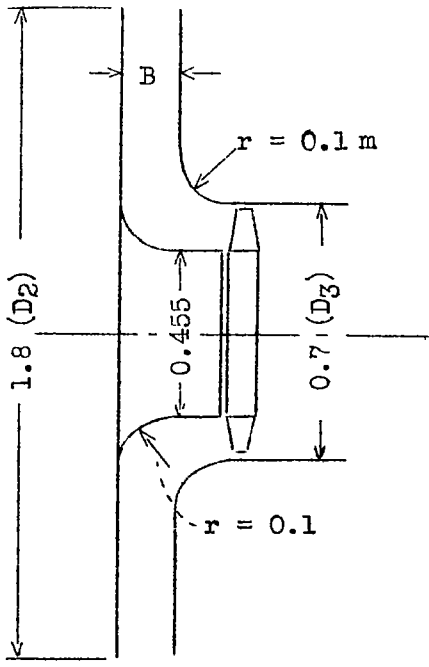


Figure 19.

Figure 22.- Sketch of axial fan with vaneless diffuser.



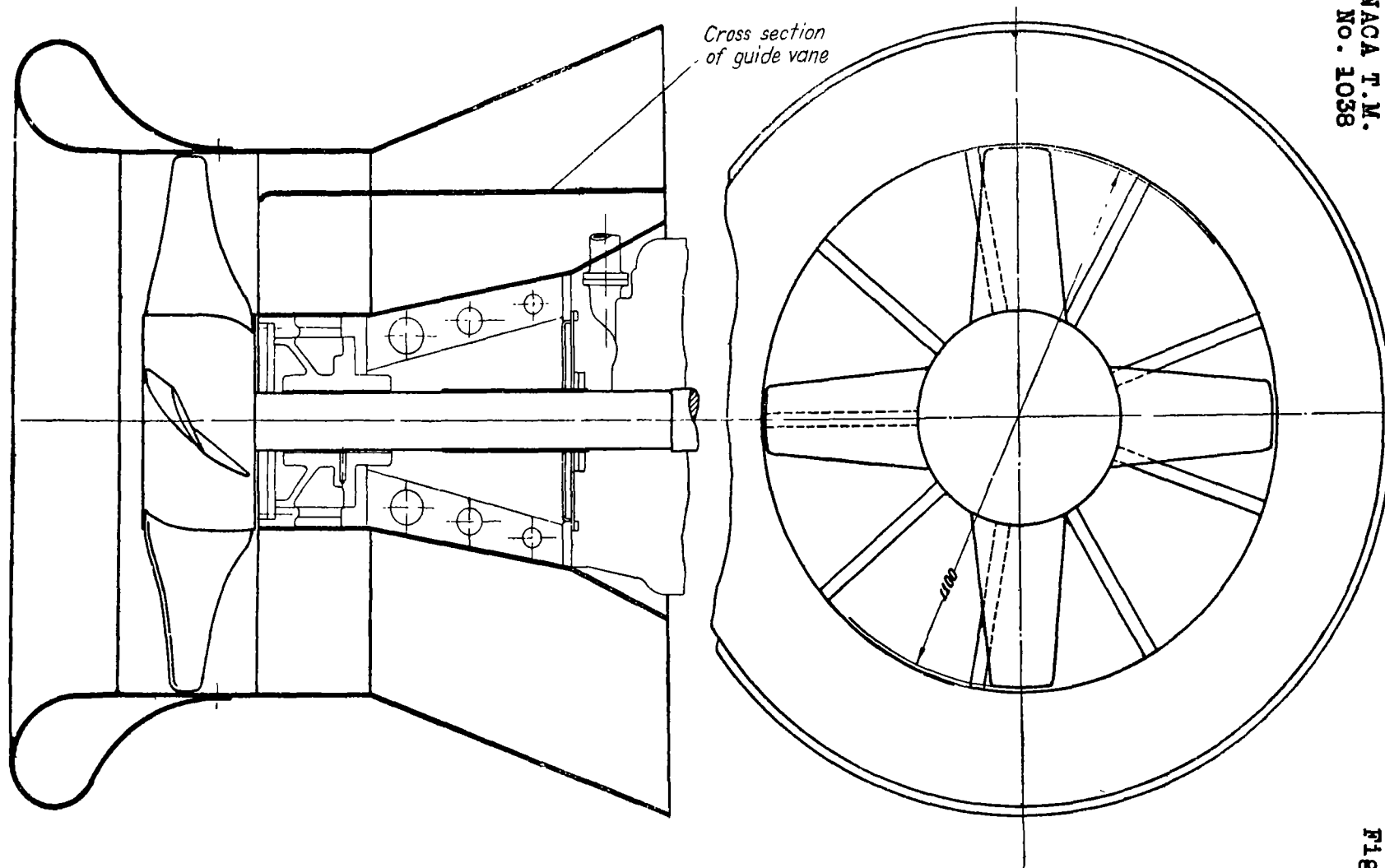


Figure 20.- Axial fan with guide vanes.

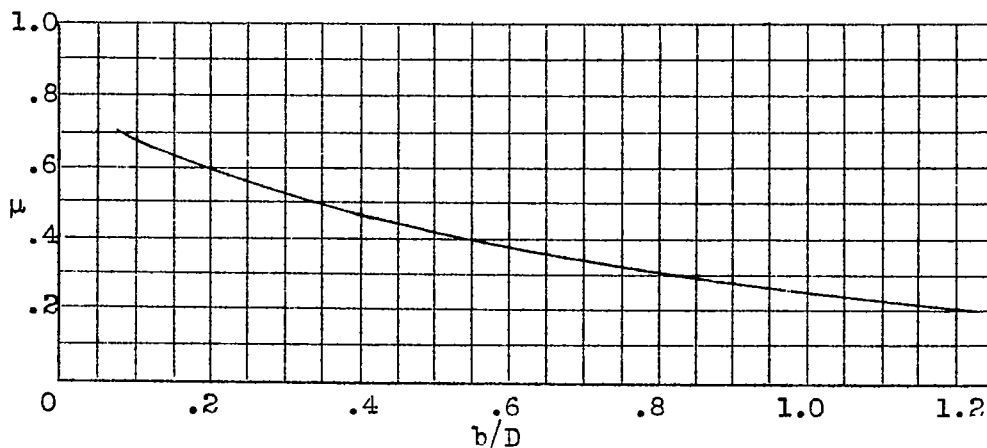


Figure 23.- Curve of μ against b/D .

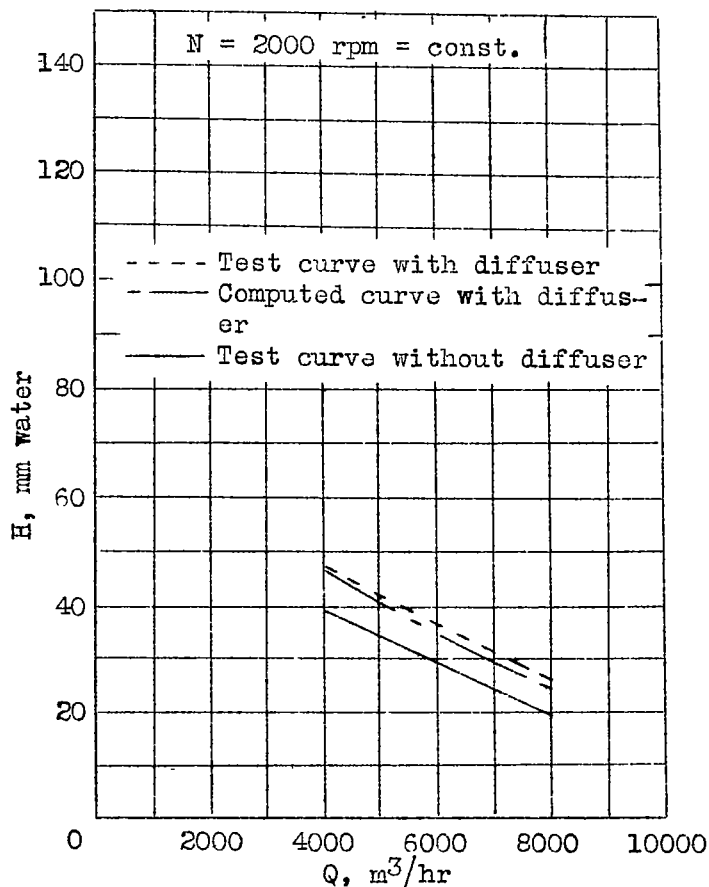


Figure 24.- Computed and test curves of H_{sta} of an axial fan of the CAHI type. ($D = 510 \text{ mm}$) with vaneless diffuser.

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